Some Examples on Ring and Fields

Ex-1 Show that the ~~[illegible]~~ set of ~~[illegible]~~ numbers ~~[illegible]~~ of the form $a\sqrt{2}$, where $a \in Q$ is not a ring under usual addition and multiplication.

Sol^n

Clearly the given set is a subset of irrational numbers.

Let S be the set.

Now $2\sqrt{2},\ 3\sqrt{2} \in S$

but $(2\sqrt{2}) \times (3\sqrt{2}) = 6 \times 2 = 12 \notin S$, since 12 is a rational number.

Hence S does not form a ~~[illegible]~~ ring under usual addition and multiplication.

Ex-2 Show that the set $\{a + b\sqrt{7} : a, b$ are integers$\}$ forms a commutative ring with respect to usual addition and multiplication.

Solution

Let $M = \{a + b\sqrt{7} : a, b \in Z\}$, where Z is the set of integers.

Let $a_1 + b_1\sqrt{7},\ a_2 + b_2\sqrt{7} \in M$, so $a_1, b_1, a_2, b_2 \in Z$

Now $(a_1 + b_1\sqrt{7}) + (a_2 + b_2\sqrt{7})$

$= (a_1 + a_2) + (b_1 + b_2)\sqrt{7} \in M$, since $a_1 + a_2,\ b_1 + b_2 \in Z$

Again $(a_1 + b_1\sqrt{7}) \cdot (a_2 + b_2\sqrt{7}) = (a_1a_2 + 7b_1b_2) + (a_2b_1 + a_1b_2)\sqrt{7} \in M$

since $a_1a_2 + 7b_1b_2,\ a_2b_1 + a_1b_2 \in Z$